IMPROVEMENT OF TILE DRAINAGE SYSTEMS USING DOUBLE MOLE DRAINS FOR LANDS UNDER ARTESIAN PRESSURE

تحسين نظم الصرف المغطى باستخدام مصارف مشكلة مزدوجة في الأراضي المعرضة لضغوط بيزومترية

By

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الخلاصة - يقدم هذا البحث نظام جديد للصرف بهدف حماية الأراضي المروية من تسرب المياه الجوفية - تحت ضغط بيزومتري من أسفل الى أعلى وذلك باستخدام نظام المصارف المشكلة المزدوجة كمصارف مساعدة للمصارف المغطاه والموجودة فعلا للتحكم في مستوى المياه الأرضية وفي هذا البحث تم التوصل الى معادلات رياضية عامة تربط بين المصارف المغطاه والمصارف المشكلة المزدوجة المساعدة - لايجاد دالة الجهد المركب (W) ودالة جهد السرعة (ϕ) ودالة السريان (Ψ) . كما تم استنتاج معادلتين جديدتين للتصرف لكل مصرف من المصارف المغطاه (QT) والمصارف المشكلة المزدوجة المساعدة (Qm) . ويناقش البحث تأثير المتغيرات المختلفة على التصرفات الناتجة من كل مصرف من المصارف المشكلة المزدوجة والمصارف المغطاه كما يوضح تلك التأثيرات بعرض مجموعة من المنحنيات التوضيحية . ويقدم البحث دراسة مقارنة بين حالة استخدام المصارف المشكلة المفردة كمصارف مساعدة للمصارف المغطاه وحالة استخدام المصارف المشكلة المزدوجة كمصارف مساعدة للمصارف المغطاه .

ABSTRACT - In some cases, agricultural clay soil is underlain by a sand and gravel aquifer of high piezometric pressure such that the clay layer may be completely water logged. Consequently, there is no possibility for plant growth. Some researchers proposed mathematical equations for designing a system of tile drains for lowering the water table. In this paper, a new improvement for this problem is presented. A solution for an existing tile drainage system by using double mole drains of the same diameter is presented. The problem is hydrodaynamically treated using the theory of complex functions and the theory of images. The complex potential, the velocity potential and the stream functions are established. New formulas for tile drains discharge as well as double mole drains discharge are derived. Functions for velocity components at a general point in the flow field are given. The effect of different variables on the discharge per mole drain and per tile drain is investigated. A comparison between a system of single mole drains and a system of double mole drains is made. Each system is considered as an assistant component for the drainage process.

INTRODUCTION

The study of the problem of tile and mole drainage systems comprises the topics of drain discharge formulae and the water table decline under the influnce of the drainage system and subsequent graphical solutions. The emphasis in this paper is on drain discharge analysis since it is the first natural step. The results we obtain here will be used in a future work to tackle the problem of water table decline and graphical solution. Moreover, drain discharge analysis requires that steady state condition has been reached and therefore the analysis in this paper considers only steady state condition.

Many researchers such as: Muskat (1), Hammad (2), Hathoot (3,4,5) Amer (6), Kirkham (7), and Hinesly (8,9) attempted the problem of draining an irrigated clay cap underlain by an artesian aquifer. This problem is related to some Egyptian lands.

Many investigators attempted problems of single or double mole drains because mole drainage is the cheapest and the simplest means for drainage irrigated lands. Mole drainage is successfully used in heavy soil. It is only applied to soils which are uniform and free from rocks. All investigators dealt with the problem of designing tile or mole drainage systems considering one system of mole or tile drains . In the present work , the problem of a tile drainage system assisted by a system of double mole drains . treated for a top clay cap underlain by an artesian aquifer.

In the proposed system a pair of mole drains is constructed above each tile drain. An additional even number of mole drain pairs are constructed in the spacing between each two adjacent tile drains. The whole mole drains array is lying in the same plane which lies above the plane of tile drains by a vertical distance equals b. The geometry of the problem and its geological section are shown in fig. (1).

MATHEMATICAL MODEL

The system of tile and double mole drains shown in Fig. (1), may be represented by point sinks located at drain centres.

The complex potential of point sinks for tile drains is (3)

$$w_1 = M. \ln \sin \pi Z/L + C_1 \qquad \ldots (1)$$

in which M is the point sink strength for tile drains, Z is the complex variable ($Z = X+iy$), $i = \sqrt{-1}$, C_1 is a real constant and L is the drain spacing .

The complex potential of the line of point sinks representing the array of double mole drains is given as follows :

$$\begin{aligned} W2 = & \; m \sum_{o}^{n} \ln \sin \pi (z - ib - (na + e)) /L + m \sum_{o}^{n} \ln \sin \pi (z - ib + (na + e))/L \\ & + m \sum_{1}^{n} \ln \sin \pi (z - ib + (na - e))/L \\ & + m \sum_{1}^{n} \ln \sin \pi (z - ib - (na - e))/L + C2 \end{aligned} \qquad \ldots (2)$$

in which m is the point sink strength for mole drains ;
a is the spacing between two successive pairs of mole drains ;
e is the distance between two mole drains in one pair ;
n equals half the number of mole drain pairs lying between two adjacent tile drains, i.e. $n = \frac{1}{2} [\frac{L - a}{a}]$; and n always has an integer value ; and
C2 is a real constant .

To represent the effect of aquifer-aquifard interface, which is an equipotential line, a similar factitious system of sources is assumed as shown in Fig. (2).

Therefore, the complex potential of point factitious sources is

$$W3 = -M. \ln \sin \pi(z + 2iD) /L + C3 \qquad \ldots (3)$$

Also, the complex potential of a factitious array of point sources representing the image of the mole drains array is given as follows :

$$\begin{aligned} W4 = & - m \sum_{o}^{n} \ln \sin \pi (z + 2iD + ib - (na + e))/L - m \sum_{o}^{n} \ln \sin \pi (z + 2iD + i D +(na + e))/L \\ & - m \sum_{1}^{n} \ln \sin \pi (z + 2iD + ib + (na - e))/L \\ & - m \sum_{1}^{n} \ln \sin \pi (z + 2iD + ib - (na -e))/L + C_4 \end{aligned} \qquad \ldots (4)$$

Therefore, the complex potential of the four systems is

$$W = W1 + W2 + W3 + W4 + C \qquad \ldots (5)$$

in which C is a real constant .
Substituting for $z = x + iy$ and simplifying :

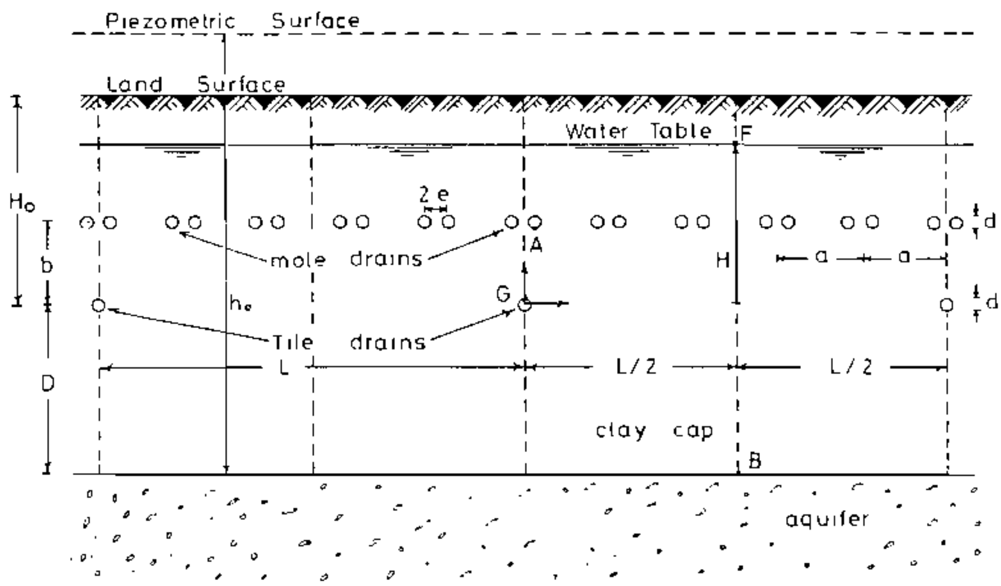


Fig.(1) Geametry of The Problem

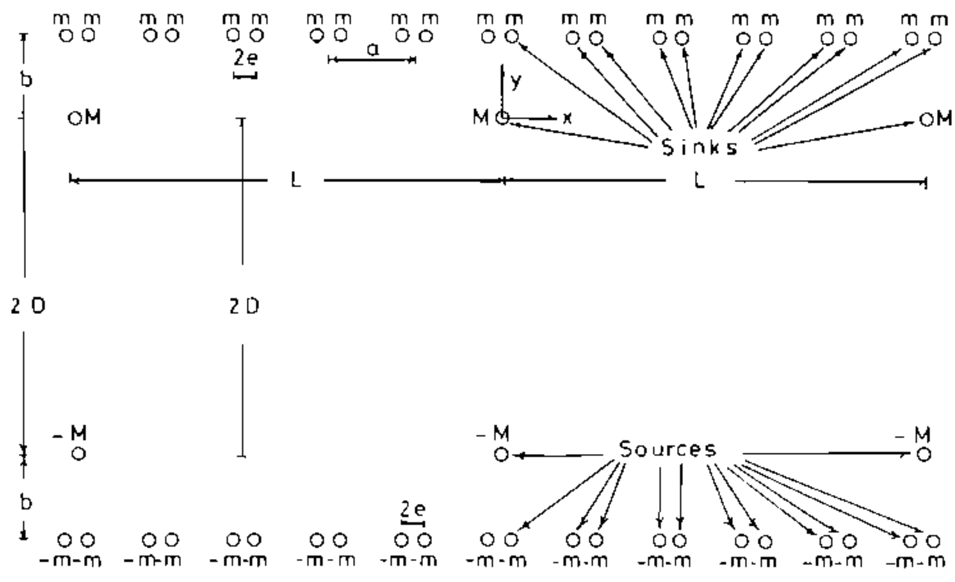


Fig (2) Mathematical Model.

$$
\begin{aligned}
W = &+ M.\ \ln \left[\sin \frac{\pi X}{L} . \cosh \frac{\pi Y}{L} + i \cos \frac{\pi X}{L} . \sinh \frac{\pi Y}{L} \right] \\
&- M.\ \ln \left[\sin \frac{\pi X}{L} . \cosh \frac{\pi (Y+2D)}{L} + i \cos \frac{\pi X}{L} . \sinh \frac{\pi (y+2D)}{L} \right] \\
&+ m \sum_{o}^{n} \ln \left[\sin \frac{\pi (X-(na+e))}{L} \cosh \frac{\pi Y1}{L} + i \cos \frac{\pi (X-(na+e))}{L} \sinh \frac{\pi Y1}{L} \right] \\
&+ m \sum_{o}^{n} \ln \left[\sin \frac{\pi (X+(na+e))}{L} \cosh \frac{\pi y1}{L} + i \cos \frac{\pi (X+(na+e))}{L} \sinh \frac{\pi Y1}{L} \right] \\
&+ m \sum_{1}^{n} \ln \left[\sin \frac{\pi (X+(na-e))}{L} \cosh \frac{\pi Y1}{L} + i \cos \frac{\pi (X+(na-e))}{L} \sinh \frac{\pi Y1}{L} \right] \\
&+ m \sum_{1}^{n} \ln \left[\sin \frac{\pi (X-(na-e))}{L} \cosh \frac{\pi Y1}{L} + i \cos \frac{\pi (X-(na-e))}{L} \sinh \frac{\pi Y1}{L} \right] \\
&- m \sum_{o}^{n} \ln \left[\sin \frac{\pi (X-(na+e))}{L} \cosh \frac{\pi Y2}{L} + i \cos \frac{\pi (X-(na+e))}{L} \sinh \frac{\pi Y2}{L} \right] \\
&- m \sum_{o}^{n} \ln \left[\sin \frac{\pi (X+(na+e))}{L} \cosh \frac{\pi Y2}{L} + i \cos \frac{\pi (X+(na+e))}{L} \sinh \frac{\pi Y2}{L} \right] \\
&- m \sum_{1}^{n} \ln \left[\sin \frac{\pi (X+(na-e))}{L} \cosh \frac{\pi Y2}{L} + i \cos \frac{\pi (X+(na-e))}{L} \sinh \frac{\pi Y2}{L} \right] \\
&- m \sum_{1}^{n} \ln \left[\sin \frac{\pi (X-(na-e))}{L} \cosh \frac{\pi Y2}{L} + i \cos \frac{\pi (X-(na-e))}{L} \sinh \frac{\pi Y2}{L} \right] \\
&+ C \qquad \ldots (6)
\end{aligned}
$$

where,

$y1 = (y - b)$ . . . (7) $\qquad$ $y2 = (y + b + 2D)$ . . . (8)

Substituting for the complex potential $w = \phi + i\Psi$: in which ϕ is the velocity potential and Ψ is the stream function and equating real to real and imaginary to imaginary on both sides of eqn (8) and simplifying yields.

$$
\begin{aligned}
\phi = &\frac{M}{2} . \ln \left[\sin^2 \pi x/L + \sinh^2 \pi y/L \right] / \left[\sin^2 \pi x/L + \sinh^2 \pi (y+2D)/L \right] \\
&+ \frac{m}{2} . \sum_{o}^{n} . \ln \left[\sin^2 \pi (x-(na+e))/L + \sinh^2 \pi (y-b)/L \right] / \left[\sin^2 \pi (x-(na+e))/L + \sinh^2 \pi (y+b+2D)/L \right] \\
&+ \frac{m}{2} . \sum_{o}^{n} . \ln \left[\sin^2 \pi (x+(na+e))/L + \sinh^2 \pi (y-b)/L \right] / \left[\sin^2 \pi (x+(na+e))/L + \sinh^2 \pi (y+b+2D)/L \right] \\
&+ \frac{m}{2} . \sum_{1}^{n} . \ln \left[\sin^2 \pi (x-(na-e))/L + \sinh^2 \pi (y-b)/L \right] / \left[\sin^2 \pi (x-(na-e))/L + \sinh^2 \pi (y+b+2D)/L \right] \\
&+ \frac{m}{2} . \sum_{1}^{n} . \ln \left[\sin^2 \pi (x+(na-e))/L + \sinh^2 \pi (y-b)/L \right] / \left[\sin^2 \pi (x+(na-e))/L + \sinh^2 \pi (y+b+2D)/L \right] + C \quad \ldots (9)
\end{aligned}
$$

$$
\begin{aligned}
\Psi = &M \left[\tan^{-1} (\cot \pi x/L . \tanh \pi y/L) - \tan^{-1} (\cot \pi x/L . \tanh \pi (y+2D)/L) \right] \\
&+ m \sum_{o}^{n} \left[\tan^{-1} (\cot \pi (x-(na+e))/L . \tanh \pi (y-b)/L) - \tan^{-1} (\cot \pi (x-(na+e))/L . \tanh \pi (y+b+2D)/L \right] \\
&+ m \sum_{o}^{n} \left[\tan^{-1} (\cot \pi (x+(na+e))/L . \tanh \pi (y-b)/L) - \tan^{-1} (\cot \pi (x+(na+e))/L . \tanh \pi (y+b+2D)/L) \right] \\
&+ m \sum_{1}^{n} \left[\tan^{-1} (\cot \pi (x-(na-e))/L . \tanh \pi (y-b)/L) - \tan^{-1} (\cot \pi (x-(na-e))/L . \tanh \pi (y+b+2D)/L) \right] \\
&+ m \sum_{1}^{n} \left[\tan^{-1} (\cot \pi (x+(na-e))/L . \tanh \pi (y-b)/L) - \tan^{-1} (\cot \pi (x-(na-e))/L . \tanh \pi (y+b+2D)/L) \right] \quad \ldots (10)
\end{aligned}
$$

VELOCITY CONSIDERATIONS

The velocity components u and v at any point in the flow field are given by (10) :

$$u = -\partial\phi/\partial x \quad \ldots (11) \quad \text{and} \quad v = -\partial\phi/\partial y \quad \ldots (12)$$

where u and v are the velocity components in the x and y directions, respectively, fig (2).

Differentiating Eq. (7) partially with respect to x and y, respectively, and simplifying we get :

$$\begin{aligned} u = & -M\pi/2L\ ((\sin(2\pi x/L)/(\sin^2\pi x/L + \sinh^2\pi y/L)) - (\sin(2\pi x/L)/(\sin^2\pi x/L + \sinh^2\pi(y+2D)/L))) \\ & - m\pi/2L \sum_{o}^{n} ((\sin(2\pi(x-(na+e))/L))/(\sin^2\pi(x-(na+e))/L + \sinh^2\pi(y-b)/L) \\ & \quad -(\sin(2\pi(x-(na+e))/L))/(\sin^2\pi(x-(na+e))/L + \sinh^2\pi(y+b+2D)/L)) \\ & - m\pi/2L \sum_{o}^{n} ((\sin(2\pi(x+(na+e))/L))/(\sin^2\pi(x+(na+e))/L + \sinh^2\pi(y-b)/L) \\ & \quad -(\sin(2\pi(x+(na+e))/L))/(\sin^2\pi(x+(na+e))/L + \sinh^2\pi(y+b+2D)/L)) \\ & - m\pi/2L \sum_{1}^{n} ((\sin(2\pi(x-(na-e))/L))/(\sin^2\pi(x-(na-e))/L + \sinh^2\pi(y-b)/L) \\ & \quad -(\sin(2\pi(x-(na-e))/L))/(\sin^2\pi(x-(na-e))/L + \sinh^2\pi(y+b+2D)/L)) \\ & - m\pi/2L \sum_{1}^{n} ((\sin(2\pi(x+(na-e))/L))/(\sin^2\pi(x+(na-e))/L + \sinh^2\pi(y-b)/L) \\ & \quad -(\sin(2\pi(x+(na-e))/L))/(\sin^2\pi(x+(na-e))/L + \sinh^2\pi(y+b+2D)/L)) \quad \ldots (13) \end{aligned}$$

and

$$\begin{aligned} V = & -M\pi/2L\ ((\sinh(2\pi y/L)/(\sin^2(\pi x/L) + \sinh^2\pi y/L) - (\sinh(2\pi(y+2D))/L/(\sin^2\pi x/L + \sinh^2\pi(y+2D)/L)) \\ & - m\pi/2L \sum_{o}^{n} ((\sinh(2\pi(y-b)/L)/(\sin^2\pi(x-(na+e))/L + \sinh^2\pi(y-b)/L) \\ & \quad -(\sinh(2\pi(y+b+2D)/L)/(\sin^2\pi(x-(na+e))/L + \sinh^2\pi(y+b+2D)/L)) \\ & - m\pi/2L \sum_{o}^{n} ((\sinh(2\pi(y-b)/L)/(\sin^2(\pi(x+(na+e))/L + \sinh^2\pi(y-b)/L) \\ & \quad -(\sinh(2\pi(y+b+2D)/L)/(\sin^2\pi(x+(na+e))/L + \sinh^2\pi(y+b+2D)/L)) \\ & - m\pi/2L \sum_{1}^{n} ((\sinh(2\pi(y-b)/L)/(\sin^2\pi(x-(na-e))/L + \sinh^2\pi(y-b)/L) \\ & \quad -(\sinh(2\pi(y+b+2D)/L)/(\sin^2\pi(x-(na-e))/L + \sinh^2\pi(y+b+2D)/L)) \\ & - m\pi/2L \sum_{1}^{n} ((\sinh(2\pi(y-b)/L)/(\sin^2\pi(x+(na-e))/L + \sinh^2\pi(y-b)/L) \\ & \quad -(\sinh(2\pi(y+b+2D)/L)/(\sin^2\pi(x+(na-e))/L + \sinh^2\pi(y+b+2D)/L)) \quad \ldots (14) \end{aligned}$$

From Eq. (11), it is evident that for $x = o$ and $x = \pm L/2$ the horizontal component of velocity u is zero. This result satisfies the boundary condition at the vertical lines of symmetry $x = o$ and $x = + L/2$. It is also clear that the horizontal velocity component, u, vanishes at $y = -D$. This result satisfies the condition that the horizontal line $y = -D$ is an equipotential line .

DISCHARGE FORMULAS

The equipotential function, ϕ, may be writen in the following form

$$\phi = K\left(\frac{p}{\rho g} + y\right) \qquad \ldots (15)$$

where K is the hydraulic conductivity of the clay cap, p is the pressure at any point at height y from the x-axis, ρ is the density of drained water and g is the acceleration due to gravity.

Applying Eqs. (9) and (15) to point B (L/2,-D), we get $C = K(ho - D)$.....(16)
where ho is the piezometric head of the lower sand and gravel aquifer.

Applying Eqs. (9) and (15) to point G (0.0,d/2) at the drain outlet, where the drain is running just full and the pressure is atmospheric and simplifying, we get

$$\begin{aligned} K(d/2+D-ho) &= M/2 \,.\, \ln \left(\sinh^2(\pi d/2L) \,/\, \sinh^2(\pi(d/2+2D)/L)\right) \\ &+ m.\sum_{0}^{n}.\ \ln \left((\sin^2 \pi(na+e)/L + \sinh^2 \pi(d/2-b)/L)/\right. \\ &\qquad \left.(\sin^2 \pi(na+e)/L + \sinh^2 \pi(d/2+b+2D)/L)\right) \\ &+ m.\sum_{1}^{n}.\ \ln \left((\sin^2 \pi(na-e)/L + \sinh^2 \pi(d/2-b)/L)/\right. \\ &\qquad \left.(\sin^2 \pi(na-e)/L + \sinh^2 \pi(d/2+b+2D)/L)\right) \end{aligned} \qquad \ldots (17)$$

Applying Eqs. (9) and (15) to point A (e,(b-d/2)) at the mole drain bottom, where the pressure is atmospheric, and simplifying, we have

$$\begin{aligned} K(b+D-ho-d/2) &= M/2 \,.\, \ln \left((\sin^2 \pi e/L + \sinh^2 \pi(b-d/2)/L)/\right. \\ &\qquad \left.(\sin^2 \pi e/L + \sinh^2 \pi(b-d/2+2D)/L)\right) \\ &+ m/2 \,.\, \ln \left(\sinh^2 \pi(d/2L) \,/\, \sinh^2 \pi(2b+2D-d/2)/L\right) \\ &+ m/2.\sum_{0}^{n} \ln \left((\sin^2 \pi(na+2e)/L + \sinh^2 \pi d/2L)/\right. \\ &\qquad \left.(\sin^2 \pi(na+2e)/L + \sinh^2 \pi(2b+2D-d/2)/L)\right) \\ &+ m/2.\sum_{1}^{n} \ln \left((\sin^2 \pi(na-2e)/L + \sinh^2 \pi(d/2L))/\right. \\ &\qquad \left.(\sin^2 \pi(na-2e)/L + \sinh^2 \pi(2b+2D-d/2)/L)\right) \\ &+ m/2.\sum_{1}^{n} \ln \left((\sin^2 \pi na/L + \sinh^2 \pi d/2L)/\right. \\ &\qquad \left.(\sin^2 \pi na/L + \sinh^2 \pi(2b+2D-d/2)/L)\right) \end{aligned} \qquad \ldots (18)$$

Applying Eqs. (9) and (15) to point F (L/2,H) on the water table , Fig. (1), we get

$$\begin{aligned} K(H+D-ho) &= M/2 \,.\, \ln \left(\cosh^2 \pi H/L \,/\, \cosh^2 \pi(H+2D)/L\right) \\ &+ m\sum_{0}^{n} \ln \left((\cos^2 \pi(na+e)/L + \sinh^2 \pi(H-b)/L)/\right. \\ &\qquad \left.((\cos^2 \pi(na+e)/L + \sinh^2 \pi(H+b+2D)/L)\right) \\ &+ m\sum_{1}^{n} \ln \left((\cos^2 \pi(na-e)/L + \sinh^2 \pi(H-b)/L)/\right. \\ &\qquad \left.(\cos^2 \pi(na-e)/L + \sinh^2 \pi(H+b+2D)/L)\right) \end{aligned} \qquad \ldots (19)$$

Eqns (17), (18) and (19) may be rewritten as follows :

$$K(d/2+D-ho) = M\delta_1 + m(\delta 2 + \delta 3) \qquad \ldots (20)$$

$$K(b+D-ho-d/2) = M\delta_4 + m(\delta 5 + \delta 6 + \delta 7 + \delta 8) \qquad \ldots (21)$$

$$K(H+D-ho) = M\,\delta 9 + m(\delta 10 + \delta 11) \qquad \ldots (22)$$

where

$$\delta 1 = 1/2 \,.\, \ln \left[(\sinh^2 \pi d/2L) \,/\, (\sinh^2 \pi (d/2+2D)/L) \right]$$

$$\delta 2 = \sum_{0}^{n} \ln \left[(\sin^2 \pi(na+e)/L + \sinh^2 \pi(d/2-b)/L) / (\sin^2 \pi(na+e)/L + \sinh^2 \pi(d/2+b+2D)/L) \right]$$

$$\delta 3 = \sum_{1}^{n} \ln \left[(\sin^2 \pi(na-e)/L + \sinh^2 \pi(d/2-b)/L) / (\sin^2 \pi(na-e)/L + \sinh^2 \pi(d/2+b+2D)/L) \right]$$

$$\delta 4 = 1/2. \ln \left[(\sin^2 \pi e/L + \sinh^2 \pi(b-d/2)/L) / (\sin^2 \pi e/L + \sinh^2 \pi(b-d/2+2D)/L) \right]$$

$$\delta 5 = 1/2. \ln \left[(\sinh^2 \pi d/2L) \,/\, (\sinh^2 \pi (2b+2D-d/2)/L) \right]$$

$$\delta 6 = 1/2. \sum_{0}^{n} \ln \left[(\sin^2 \pi(na+2e)/L + \sinh^2 \pi d/2L) / (\sin^2 \pi (na+2e)/L + \sinh^2 \pi (2b+2D-d/2)/L) \right]$$

$$\delta 7 = 1/2. \sum_{1}^{n} \ln \left[(\sin^2 \pi(na-2e)/L + \sinh^2 \pi d/2L) / (\sin^2 \pi(na-2e)/L + \sinh^2 \pi(2b+2D-d/2)/L) \right]$$

$$\delta 8 = \sum_{1}^{n} \ln \left[(\sin^2 \pi na/L + \sinh^2 \pi d/2L) / (\sin^2 \pi na/L + \sinh^2 \pi(2b+2D-d/2)/L) \right]$$

$$\delta 9 = 1/2. \ln \left[\cosh^2 \pi H/L \,/\, \cosh^2 \pi (H+2D)/L \right]$$

$$\delta 10 = \sum_{0}^{n} \ln \left[(\cos^2 \pi (na+e)/L + \sinh^2 \pi (H-b)/L) / (\cos^2 \pi (na+e)/L + \sinh^2 \pi (H+b+2D)/L) \right]$$

$$\delta 11 = \sum_{1}^{n} \ln \left[(\cos^2 \pi (na-e)/L + \sinh^2 \pi(H-b)/L) / (\cos^2 \pi (na-e)/L + \sinh^2 \pi(H+b+2D)/L) \right]$$

From Eqs. (20) and (22), yields

$$K(d/2+H+2D-2ho) = M(\delta 1 + \delta 9) + m\,(\delta 2+\delta 3+\delta 10+\delta 11) \quad \ldots (23)$$

Also, from Eqs. (21) and (22), we get

$$K(H + 2D + b - 2ho - d/2) = M\,(\delta 4 + \delta 9) + m\,(\delta 5 + \delta 6 + \delta 7 + \delta 8 + \delta 10 + \delta 11) \quad \ldots (24)$$

Eqns (23) and (24) may be rewritten in this form

$$K\gamma 1 = M\eta 1 + m\eta 2 \quad \ldots (25)$$

$$K\gamma 2 = M\eta 3 + m\eta 4 \quad \ldots (26)$$

where

$$\gamma 1 = (d/2 - H + 2D - 2ho)$$

$$\gamma 2 = (H - 2D + b - d/2 - 2ho)$$

$$\eta 1 = \delta 1 + \delta 9$$

$$\eta 2 = \delta 2 - \delta 3 + \delta 10 + \delta 11$$

$\eta 3 = \delta 4 + \delta 9$

$\eta 4 = \delta 5 + \delta 6 + \delta 7 + \delta 8 + \delta 10 + \delta 11$

$\therefore$ From Eqs. (25) and (26) and solving for M and m we get,

$$M = K\ \Theta \quad \ldots (27)$$
$$m = K\ (\gamma 1 - \eta 1\ \Theta) / \eta 2 \quad \ldots (28)$$

where $\Theta = (\gamma 1 . \eta 4 - \gamma 2 . \eta 2) / (\eta 1 . \eta 4 - \eta 2 . \eta 3)$

Therefore, from Eq. (27), the discharge reaching each unit length of tile drain is given by

$$Q T = 2 \pi K \Theta \quad \ldots (29)$$

Also, from Eq. (28), the discharge reaching each unit length of mole drain is given by

$$Qm = 2 \pi K (\gamma 1 - \eta 1 . \Theta) / \eta 2 \quad \ldots (30)$$

EFFECT OF DIFFERENT PARAMETERS ON THE DISCHARGE OF TILE AND MOLE DRAINS.

From Eqs. (29) and (30), it is evident that the discharge per unit length of tile drain or mole drain is dependent on certain parameters such as b, L, e, a, D, ho, L and H

The effect of each parameter on the unit discharge of tile drain, QT, and the unit discharge of mole drain, Qm, has been studied as follows

In practice, the depths of tiles, moles and water table depend on the drainage depth, where the drainage depth is assumed to be the difference between the land surface and the water table. Hence, parameters such as depths of tiles, moles and water table do not appear explicity in the analysis.

A. EFFECT OF VERTICAL SPACING, b.

In Fig. (3) the effect of vertical spacing on drain discharge is illustrated, where b/d is shown plotted against Q_T/ K.d and Q_m/K.d. When the vertical spacing ratio b/d increases the unit discharge of tile drain increases but the unit discharge of mole drain decreases. This is because increasing b means decreasing water head for mole drains.

B. EFFECT OF WATER HEAD, H.

Fig. (4) shows that the unit discharge of both tile and mole drains decreases when the water head H increases. This is due to the fact that increasing H means a smaller required depression of water table from the original water surface(4).When the water head ratio, H/d, is doubled, from 10 to 20 the discharge percentge decrease in tile drain is 6 per cent and the discharge percentage decrease in mole drain is 16.5 per cent. From this, it is clear that the effect of increasing H on the discharge is small.

C. EFFECT OF TILE DRAIN SPACING, L.

Fig.(5) illustrates the effect of tile drain spacing ,L, on the discharge per unit length of both tile drain and mole drain. Inspecting Fig.(5),it is noted that unit discharge of both tile drain and mole drain increases when increasing tile drain spacing L.

D. EFFECT OF THE PIEZOMETRIC HEAD OF THE AQUIFER, ho.

The effect of piezometric head of the aquifer, ho, on the discharge per unit length of both tile drain and mole drain is illustrated in Fig.(6). From this figure, it is evident that the discharge per unit length of both tile drain and mole drain increases on increasing the piezometric head, ho. This is due to the fact that increasing ho means a more quantity of water has to be drained . When the piezometric head ratio ho/D is increased by 25%, from ho/D = 1.6 to ho/D = 2.0, the discharge percentage increase for tile drain is 35% and the discharge percentage increase for mole drain is 145%. This shows that, the piezometric head of the aquifer is of considerable effect on the discharge of both tile drain and mole drain

E- EFFECT OF DRAIN HEIGHT ABOVE THE AQUIFER-AQUIFARD INTERFACE,D

Figs. (7) and (8) illustrate the effect of drain height above the aquifer-aquifard interface on the discharge per unit length of both tile drain and mole drain. Figs. (7) and (8) show the relation between the discharge per unit length of both tile drain and mole drain and soil thickness D, for the case of constant piezometric head ho and constant pressure head (ho-D), respectively.

It is evident from Figs. (7) and (8) that, the discharge per unit length of both tile drain and mole drain decreases as the drain height above the aquifer-aquifard interface, D, increases because the effective head on the aquifer increases. Fig.(7) shows that, the discharge percentage decrease for tile drain is 62.6% while it is 88.4% for mole drain when the soil thickness ratio D/H is increased from 2 to 5. Also, from Fig.(8), it is clear that, the discharge percentage decrease for tile drain is 28.2% while it is 63.8% for mole drain when the soil thickness ratio D/H is increased from 2 t0 5.

From Figs(7) and (8), it is evident that, the discharge of mole drain is more affected by increasing soil thickness than the discharge of tile drain. Also, the discharge of both tile drain and mole drain is affected more by increasing soil thickness for case of constant piezometric surface height above tile drain.

F- EFFECT OF MOLE INTERNAL SPACING, 2e.

Fig. (9) shows the effect of mole internal spacing on the discharge per unit length of both tile drain and mole drain. When internal spacing ratio increases the discharge per unit length of mole drain increases. In the mean time, the discharge per unit length of tile drain decreases. As internal spacing ratio is increased from 4 to 16, the discharge percentage decrease for tile drain is 4.57% while the discharge percentage increase for mole drain is 7.04% .

G- EFFECT OF MOLE DRAIN PAIRS SPACING, a.

The effect of mole drain spacing, a, is illustrated in Fig.(10). From this figure, it is evident that, when mole drain spacing decreases the discharge per unit length of both tile drain and mole drain decreases. When mole drain spacing is decreased from 25 m to 5 m (N = 3 to 15) the total discharge for the system of mole drains and tile drains is increased by 31.39% .

Comparative Study with a System of Tile Drains which is Assisted by a System of Single Mole Drains

In the following, we shall compare the discharge of a tile drainage system assisted by a system of double mole drains with a tile drainage system assisted by a system of single mole drains. The comparative study is needed to show the advantages of double drains. In a previous paper (11) the author established the following discharge formulas for a system of tile drains assisted by a system of single mole drains

$$\grave{Q}_T = 2 \pi k \grave{\theta} \quad \ldots (31) \qquad \grave{Q}_m = 2 \pi k(\grave{\vartheta}_1 - \grave{\eta}_1 \cdot \grave{\theta}) / \grave{n}_2 \quad \ldots (32)$$

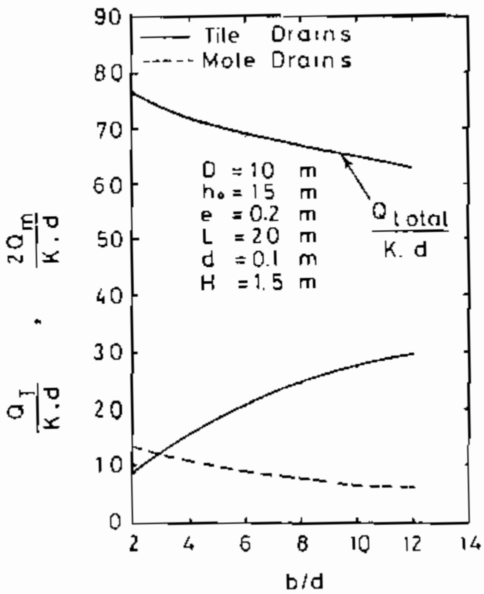
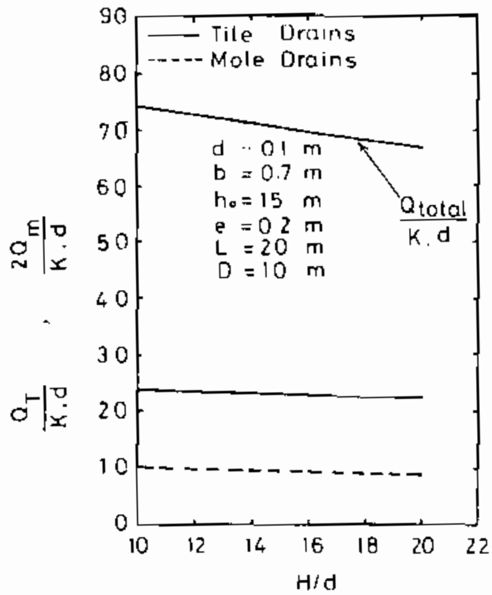


Fig.(3): Discharge Ratio Versus Vertical Spacing Ratio



Fig(4) Discharge Ratio Versus Water Head Ratio.

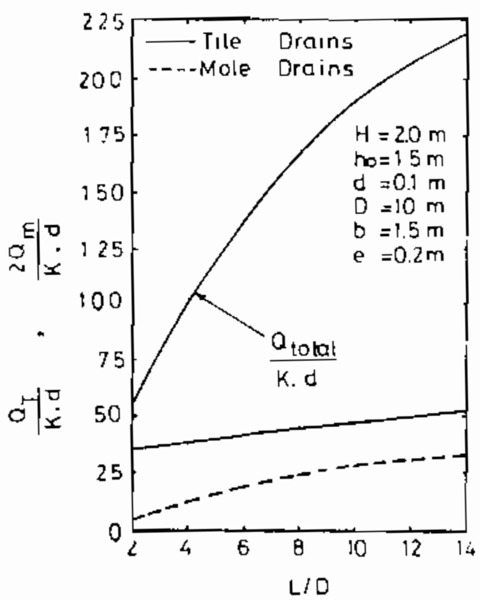


Fig.(5): Discharge Ratio Versus Horizental Spacing Ratio

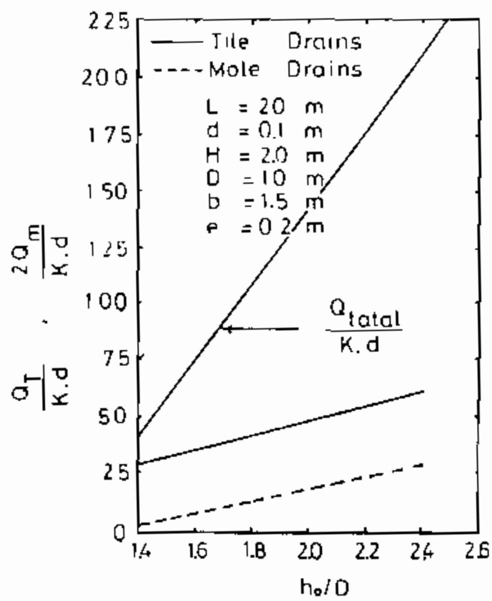


Fig.(6) Discharge Ratio Versus Piezometric Head Ratio

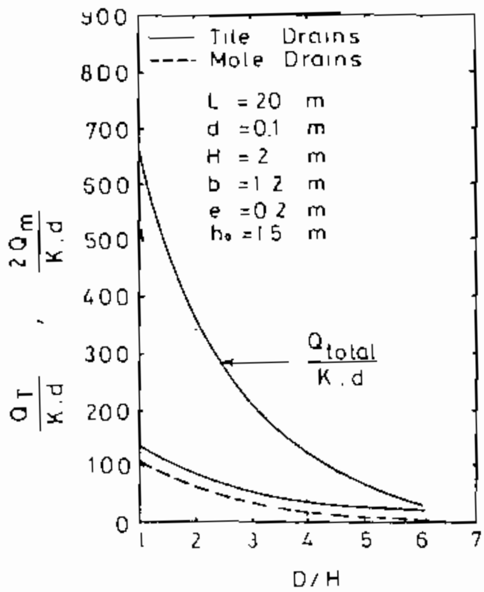


Fig (7) Discharge Ratio Versus D/H. Ratio.(h₀ = constant)

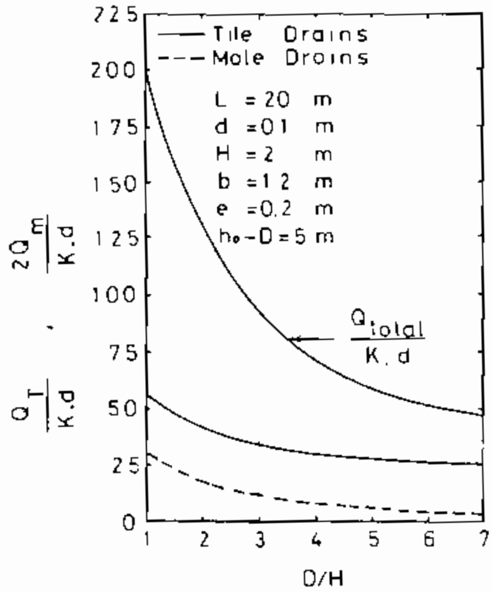


Fig (8): Discharge Ratio Versus D/H. Ratio (h₀ − D = constant).

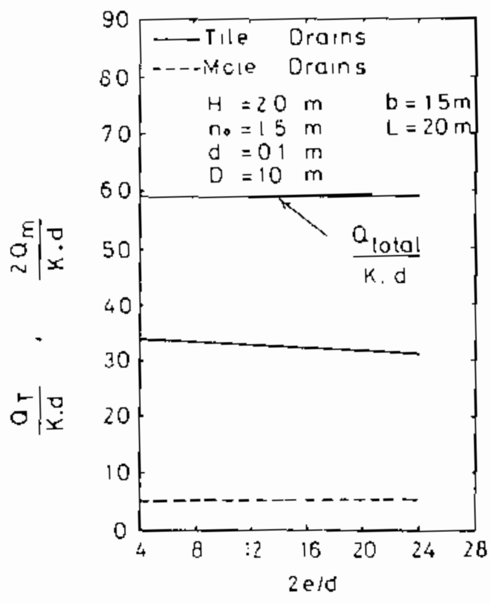


Fig (9): Discharge Ratio Versus Mole Internal Spacing Ratio.

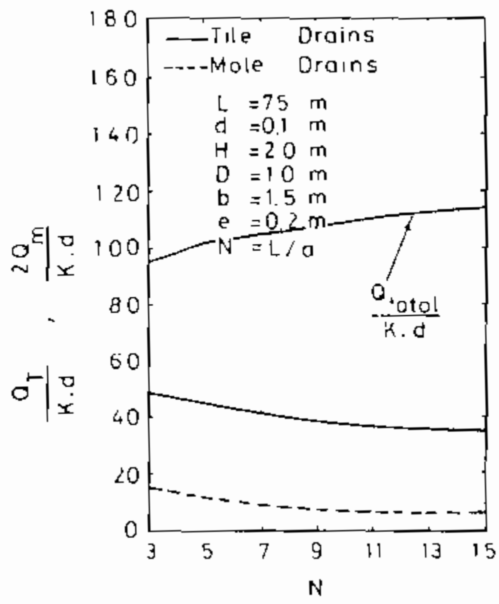
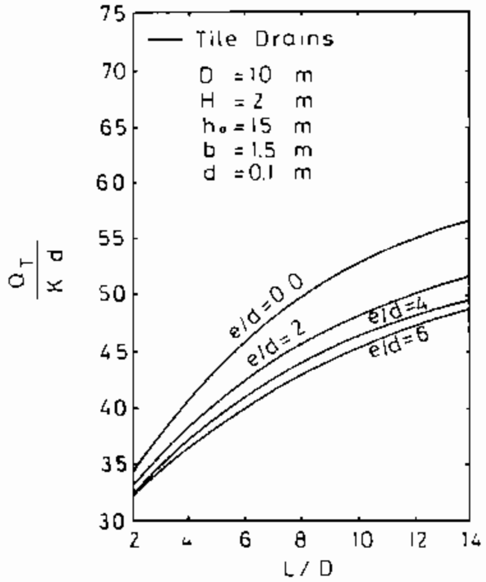
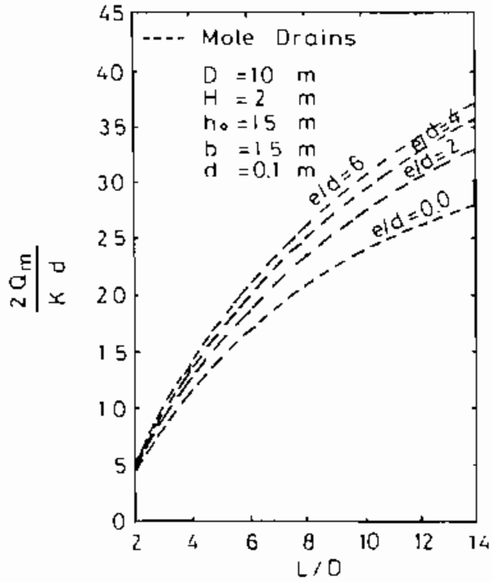


Fig (10). Discharge Ratio Versus Number of Pairs of Double Mole Drains.



Fig(11) Discharge Ratio of Tile Drains Versus Spacing Ratio,



Fig(12) Discharge Ratio of Mole Drains Versus Spacing Ratio.

where

Q'_T = the discharge reaching each unit length of tile drain.

Q'_m = the discharge reaching each unit length of mole drain

$\Theta' = (\sigma'_1 . \eta'_4 - \sigma'_2 . \eta'_2) / (\eta'_1 . \eta'_4 - \eta'_2 . \eta'_3)$

$\sigma'_1 = (d/2 + H + 2D - 2ho)$, $\sigma'_2 = (H + 2D + b - 2ho - d/z)$;

$\eta'_1 = \delta'_1 + \delta'_7$, $\eta'_2 = \delta'_2 + \delta'_3 + \delta'_8 + \delta'_9$;

$\eta'_3 = \delta'_4 + \delta'_7$ $\eta'_4 - \delta'_5 + \delta'_6 + \delta'_8 + \delta_9$

$\delta'_1 = 1/2 . \ln (\sinh^2 \pi d/2 L / \sinh^2 \pi (d/2 + 2D) / L)$

$\delta'_2 = 1/2 . \ln (\sinh^2 \pi (d/2 - b) / L / \sinh^2 \pi (d/2 + b + 2D) / L)$

$\delta'_3 = \sum_{1}^{2} . \ln ((\sin^2 \pi na/L + \sinh^2 \pi (d/2 - D) / L) / (\sin^2 \pi na/L + \sinh^2 \pi$
$(d/2 + b + 2D) / L))$

$\delta'_4 = 1/2 . \ln (\sinh^2 \pi (b - d/2) / L / \sinh^2 \pi (b + 2D - d/2) / L)$

$\delta'_5 = 1/2 . \ln (\sinh^2 \pi (-d/2) / L / \sinh^2 \pi (2b + 2D - d/2) / L)$

$\delta'_6 = \sum_{1}^{n} . \ln ((\sin^2 \pi ha/L + \sinh^2 \pi (-d/2) / L) K \sin^2 \pi na/L + \sinh^2 \pi$
$(2b + 2D - d/2) / L))$

$$\delta'_7 = 1/2 \cdot \ln (\cosh^2 \pi H / L + \sinh^2 \pi (H + 2D) / L)$$

$$\delta'_8 = 1/2 \cdot \sum_{0}^{n} \cdot \ln ((\cos^2 \pi\, na/L + \sinh^2 \pi (H - b) / L) / (\cos^2 \pi\, na/L + \sinh^2 \pi (2D + H + b) / L))$$

$$\delta'_9 = 1/2 \sum_{1}^{n} \cdot \ln ((\cos^2 \pi\, na/L + \sinh^2 \pi (H - b) / L) / (\cos^2 \pi\, na / L + \sinh^2 \pi (2D + H + b) / L))$$

Figs. (11) and (12) show the relation between the discharge ratio per unit length of both tile drain and mole drain and spacing ratio for different values of internal spacing ratio. From figs. (11) and (12) it is clear that, for small spacings e.g. (L/D = 2), the discharge taken by tile drains assisted by a system of double mole drains is smaller than the discharge taken by tile drains assisted by a system of single mole drains by about 6.5% . On the other hand, the discharge taken by one pair of double mole drains is greater than the discharge taken by a single mole drain by about 14.3% . Moreover, for large spacings e.g. (L/p = 14) the percentage decrease of the discharge for tile drains reaches about 14%, and the percentage increase of the discharge for mole drains reaches about 33.6%. From Figs. (11) and (12), it is seen that using double mole drains increases the discharge for the drainage system by about 20% with respect to that obtained when using single mole drains for large spacings.

It may be aryued that small values of old approch the case of single mole with a bigger diameter while big values simply represent the case of single mole drain. However, in practices the range of e/d in operation is 0.2 to 0.3, which represents the cost for a single mole drain under the same conditions.

Conclusions

In this paper, a new hydrodynamical analysis for a tile drainage system assisted by double mole drains is presented for the case of a clay layer, underlain by an artesian aquifer. The complex potential, the stream function and the velocity potential have been derived. Also, the velocity components at a general point in the flow field have been established. The hydrodynamical analysis is verified by applying the velocity components and boundary conditions found satisf. The new exact discharge formulas for both tile drains and mole drains are concluded. A study of the effect of different parameters such as vertical spacing, b, mole internal spacing, 2e, the piezometric head, ho water head, H, tile drain spacing, L, the drain height above the aquifer-aquifard interface, D, and mole drain spacing, a, on the discharge per, unit length of both tile drain and mole drain has been presented. From this study, it is clear that, using double mole drains increases the discharge by 20% with respect to that obtained by using single mole drains.

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NOTATIONS

The following symbols have been adopted for use in this paper :

- a = spacing between two successive pairs of mole drains;
- b = vertical spacing between lines of tile drains and mole drains;
- d = drain diameter for both tile and mole drains;
- D = depth of clay layer below tile drains;
- 2e = distance between two mole drains in one pair;
- g = acceleration due to gravity;
- ho = piezometric head of sand and gravel aquifer;
- H = height of water table above tile drains at the mid point between two successive tile drains;
- H_o = height of land surface above tile drains;
- i = $\sqrt{-1}$;
- k = Hydraulic conductivity of clay;
- L = spacing between two successive tile drains;
- m = strength of a point sink for mole drains;
- M = strength of a point sink for tile drains;
- n = ((L/a) - 1) / 2 ;
- P = pressure at any general point (x,y) ;
- Q_m = discharge reaching each unit length of mole drains ;
- Q_T = discharge reaching each unit length of tile drains ;
- u = velocity component in the x - direction ;
- V = velocity component in the y - direction ;
- w = complex potential = $\phi + i\psi$;
- x,y = coordinates of any point in the field of motion ;
- z = complex number = x + iy ;
- ϕ = the equipotential function ;
- ψ = the stream function ; and
- ρ = water density